Gradient Wind

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1 Introduction to the Gradient Wind

The equation for the gradient wind is given by

$$\frac{V^2}{R} + fV = -\frac{1}{\rho}\frac{\partial p}{\partial n}$$

where V is velocity of the flow, R is the radius of curvature of the flow, and $\hat{n}$ is the unit normal to the flow. (We are working in natural coordinates.) The equation for geostrophic flow is

$$fV_g = -\frac{1}{\rho}\frac{\partial p}{\partial n}$$

where V_g is geostrophic velocity. So we can make a substitution to get

$$\frac{V^2}{R} + fV = fV_g$$

We divide everything by f^2R to get

$$\frac{V^2}{f^2R^2} + \frac{fV}{f^2R} = \frac{fV_g}{f^2R}$$

or

$$\left(\frac{V}{fR}\right)^2 + \left(\frac{V}{fR}\right) = \left(\frac{V_g}{fR}\right)$$

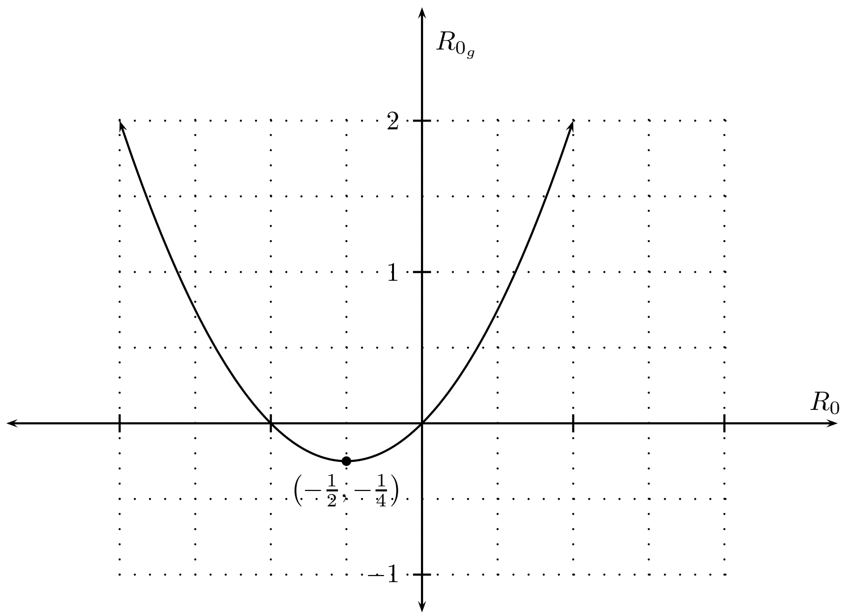
We define the Rossby Number R_0 to be

$$R_0 = \frac{V}{fR}$$

Substituting this, we get

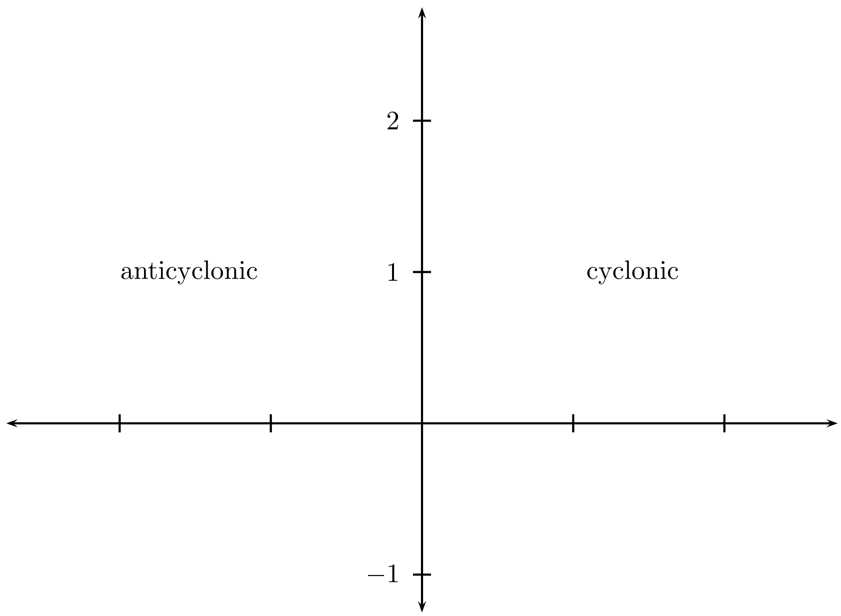
$$R_0^2 + R_0 = R_{0_g}$$

where R_{0_g} is the geostrophic Rossby Number. This is the equation for a parabola, with R_0 as the independent variable. R_{0_g} is a parameter whose value determines the number of solutions to this equation. If we plot this with R_0 as the abcissa and R_{0_g} as the ordinate, we get the parabola



2 First Division

This allows us to divide the plane into regimes. We first divide the plane into two regions: $R_0 > 0$ and $R_0 < 0$. We begin with the case $R_0 > 0$. Since $V > 0$ always (by definition), $R_0 > 0$ exactly means $fR > 0$, which is the definition of cyclonic flow. Similarly, $R_0 < 0$ defines anticyclonic flow. Then we have



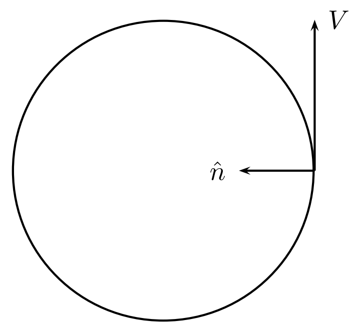
3 Second Division

We now divide the plane into the regions $R_{0_g} > 0$ and $R_{0_g} < 0$. If $R_{0_g} > 0$, then by definition

$$-\frac{1}{f^2 R}\frac{1}{\rho}\frac{\partial p}{\partial n} > 0$$

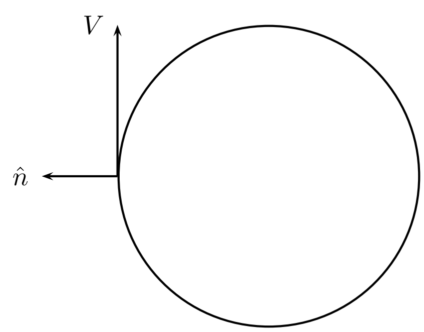
f^2 and ρ are always positive, so this means $\frac{1}{R}\frac{\partial p}{\partial n} < 0$. We can divide this up into two cases.

Case 1: $R > 0$. Then $\frac{\partial p}{\partial n} < 0$, so p decreases with n. We can summarize this with a picture:



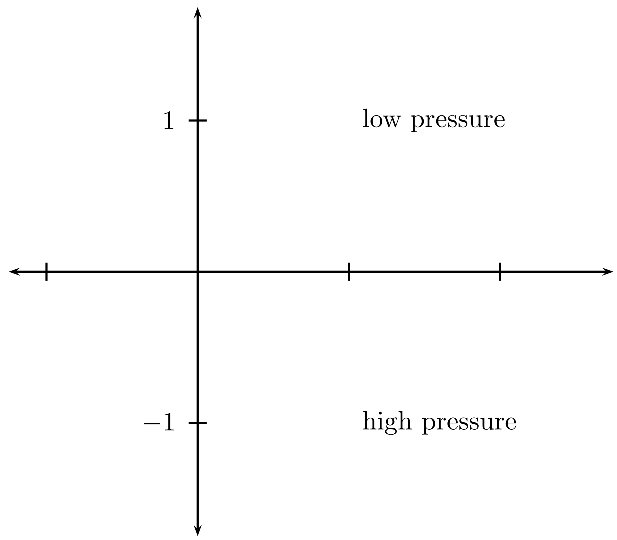
$R > 0$, which means R curves in the direction of $\hat{n}$. p is decreasing with $\hat{n}$, which describes a low pressure center.

Case 2: $R < 0$.



$R < 0$, which means R curves away from $\hat{n}$. p is now increasing with $\hat{n}$, which again describes a low pressure center.

Using a similar argument for the bottom half of the plane $\left(R_{0_g} < 0\right)$, we get



4 Third Division

Now we analyze the difference between normal and anomalous flow. This effectively translates into an argument about absolute angular momentum (M). Normal cases are defined as $M > 0$ in the Northern Hemisphere ($f > 0$) and $M < 0$ in the Southern Hemisphere ($f < 0$).

For the circularly symmetric motion described by gradient flow,

$$M = VR + \frac{fR^2}{2}$$

In the Northern Hemisphere, normal flow is $M > 0$, or

$$VR + \frac{fR^2}{2} > 0 \Longrightarrow VR > -\frac{fR^2}{2} \Longrightarrow \frac{VR}{fR^2} > -\frac{1}{2} \text{ (because } f > 0\text{)}$$

$$\Longrightarrow \frac{V}{fR} > -\frac{1}{2} \Longrightarrow R_0 > -\frac{1}{2}$$

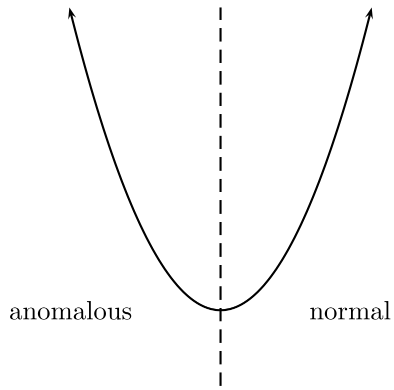
So normal flow in the Northern Hemisphere is characterized by $R_0 > -\frac{1}{2}$, and anomalous flow is characterized by $R_0 < -\frac{1}{2}$.

In the Southern Hemisphere, normal flow is $M < 0$, or

$$VR + \frac{fR^2}{2} < 0 \Longrightarrow VR < -\frac{fR^2}{2} \Longrightarrow \frac{VR}{fR^2} > -\frac{1}{2} \text{ (because } f < 0\text{)}$$

$$\Longrightarrow \frac{V}{fR} > -\frac{1}{2} \Longrightarrow R_0 > -\frac{1}{2}$$

which yields the same results as the Northern Hemisphere. Then we can divide up the plane again:



5 Eliminating Cases

This helps us to characterize normal and anomalous flows in association with cyclonic and anticyclonic flow.

	cyclonic	anticyclonic
normal	$R_0 > 0$	$-1/2 < R_0 < 0$
anomalous	no solutions	$R_0 < -1/2$

The "no solutions" box indicates that there is no overlap between anomalous flow $\left(R_0 < -\frac{1}{2}\right)$ and cyclonic flow $(R_0 > 0)$.